

Multiplication

$$12 \times 7 = 84$$

factor \times factor = product

Concrete Pictorial Abstract

referred to as CPA

The concrete-pictorial-abstract approach, based on research by psychologist Jerome Bruner, suggests that there are three steps (or representations) necessary for pupils to develop understanding of a concept. Reinforcement is achieved by going back and forth between these representations.

Concrete representation

The enactive stage - a student is first introduced to an idea or a skill by acting it out with real objects. In division, for example, this might be done by separating apples into groups of red ones and green ones or by sharing 12 biscuits amongst 6 children. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding

Pictorial representation

The iconic stage - a student has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. In the case of a division exercise this could be the action of circling objects

Abstract representation

The symbolic stage - a student is now capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$ This is the ultimate mode, for it "is clearly the most mysterious of the three."

Use a range of 'concrete objects' / 'tools to think with'

Begin with real life situations so that children use objects/'tools to think with' to represent the situation. Initially these need to be the objects in the question. Later the objects can be representative.

Using real life questions and modelling using 'concrete objects' as 'tools to think with' will ensure that children develop their **visualisation skills** (mentally thinking in your head).

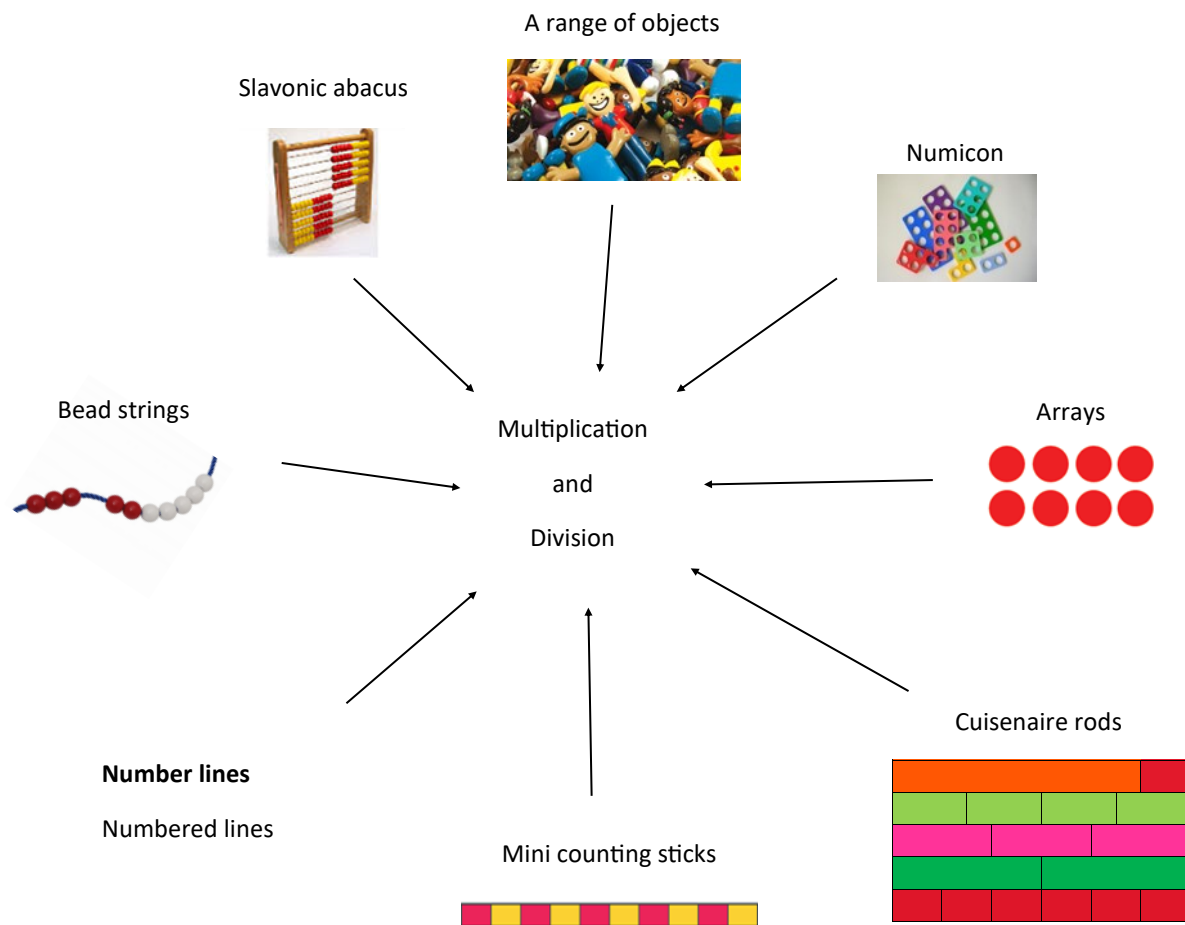
Visualisation is a key intellectual competency—it supports pupil's skills in 'seeing'/visualising what a questions is asking of them. It is very important to develop visualisation so that pupils have this skill as they move from the concrete, to pictorial and then to the abstract.

The development of visualisation skills requires a focussed approach. Visualisation needs to be discussed with the children through the use of questions i.e. What are you picturing in your head?

The importance of visualisation, in terms of how visualisation will support their learning now and in the future, needs to be shared with the pupils.

If, when solving a problem, the focus is on the answer only, then the pupils may not be exposed to the opportunity to develop their ability to visualise, thus restricting their ability to move from concrete representation (enactive) to pictorial representation (iconic) and then to abstract representation (symbolic).

'Tools to think with'



Learning

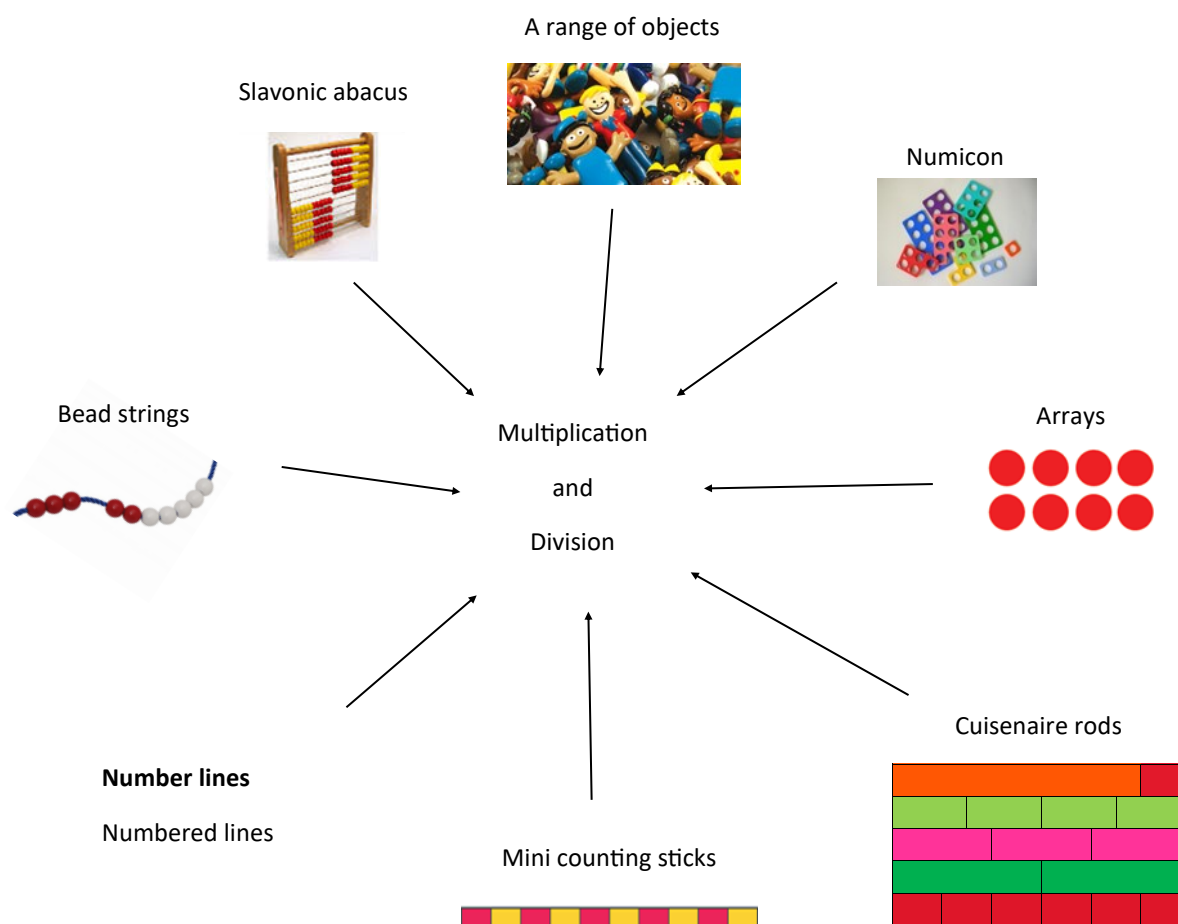
Learning and teaching are interrelated: one does not occur without the other. Genuine teaching is not linear. It is messy, arrived at by many paths, and characterised by different-sized steps and shifts in direction. Genuine teaching is directed towards landmarks and horizons.... As we learn we construct. (Fosnot and Dolk 2001, p.30)

Word problems V context problems

Word problems are often designed with little context they are usually nothing more than superficial, camouflaged attempts to get children to do the procedures teachers want them to do.

Context problems are connected as closely as possible to children's lives, rather than to 'school mathematics' (Fosnot and Dolk 2001p. 26).

'Tools to think with'



Counting in multiples of (step counting) Use the Concrete - Pictorial - Abstract approach

When counting in multiples of/step counting - it is important to ensure that the counting is modelled using real-life objects and concrete representations (cardinal) for example Numicon as well as counting sticks and a number line (ordinal).

Year 1: Count in twos, fives, and tens	Use socks, shoes, hands, gloves, Numicon...
Year 2: Count in steps of 2, 3 and 5 from 0, and in tens from any number, forward or backward	Use Numicon, counting sticks and number lines
Year 3: Count from 0 in multiples of 4, 8, 50 and 100	Use Numicon, counting sticks and number lines
Year 4: Count in multiples of 6, 7, 9 and 1000	Use Numicon, counting sticks and number lines
Year 5: Count forwards or backwards in steps of powers of 10 for any give number up to 1 000 000	Use counting sticks and number lines

Big ideas in multiplication

Reception

Unitising

The importance of Unitising

“When children are attempting to understand “how many” (how many cakes there are in the box, for example), initially children use counting. They tag and count each object once and once only.



But counting in ones is not multiplication.

The understanding that four can simultaneously be one, one row, or one bag of four cakes - is the big idea of **unitising**.

Many researchers have described -

“ how a number must be treated differently when it is unitised, a difficult idea for children”.

Prior to constructing this idea, number is used to represent single units - six represent six cookies.

To understand that this group can be counted simultaneously as one, requires a higher-order treatment of number in which groups are counted as well as the objects in the group.”

When children can **unitise** they start to use the language of unitising, for example “*five fours*”

This language demonstrates the unitising of units.

The child is counting groups not objects.

“The whole is thus seen as a number of groups of a number of objects - for example four groups of six, or 4×6 . The parts together become the new whole, and the parts (the groups) and the whole can be considered simultaneously.

- The relationship of these to the whole explain the reciprocal relationship between division and multiplication.
- Because we know the parts (the number of objects in each group and the number of groups), we can figure out the whole.
- If we know the whole and one part (the number of groups, say) we can figure out the other part (the number of objects in the group).

Unitising is also a central organising idea in mathematics because it underlies the understanding of place value: ten objects are one ten.”

Multiplication - a binary operation

“Addition and subtraction can be thought of as the joining of sets, multiplication is about replication. Addition and subtraction are **unary operations** with each input representing the same kind of element – 3 blocks added to 4 blocks or 3 oranges added to 4 oranges.

However, we need to view multiplication as a **binary operation** with two distinctive inputs (Anghileri, 2000). The first input represents the size of a set (say the number of oranges in a particular set) and the second represents the number of replications of that set (how many sets of oranges). In this way, the two numbers represent distinct elements of the multiplication process”.

Barmby, P. and Harries, T. and Higgins, S. and Suggate, J. (2009) ‘The array representation and primary children’s understanding and reasoning in multiplication.’, *Educational studies in mathematics*, 70 (3). pp.217-241.

Unary representations



Figure 2: ‘Groups of’ representation of multiplication, showing 6×7 and 7×6

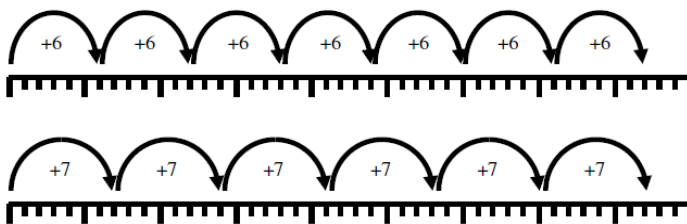


Figure 3: Number line representation of multiplication, showing 6×7 and 7×6

Binary representations

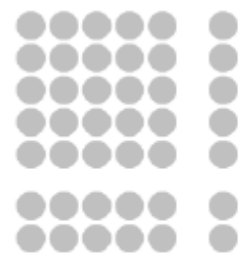


Figure 4: Array representation for 6×7 or 7×6

“Figures 2 and 3 show representations that encourage **unary** thinking and also encourage a repeated addition method of calculation – a method that whilst it may work becomes increasingly inefficient as the value of the numbers increase. These representations, which encourage repeated addition, are also problematic when the multiplication involves two rational numbers (fractions, negative numbers).

Both representations illustrate the idea of equal groups, and the number line provides help with the calculation.

Neither representation however illustrates the two important aspects of multiplication, [] namely commutativity and the distributive characteristic.

For example, when the numbers are swapped in the above diagrams, the representations will look quite different. It is not immediately obvious why the commutative law should apply. []The array representation, shown below in Figure 4, encourages pupils to develop their thinking about multiplication as a **binary** operation with rows and columns representing the two inputs”.

Barmby, P. and Harries, T. and Higgins, S. and Suggate, J. (2009) ‘The array representation and primary children’s understanding and reasoning in multiplication.’, *Educational studies in mathematics*, 70 (3). pp.217-241.

Understanding arrays as a big idea

Michael Battista et al 1998, identified four stages that children go through as they develop the ability coordinate rows and columns.

Initial stage: students structure arrays as one-dimensional paths.

They draw or fill a 3 x 6 array in an unidirectional way - perhaps by filling/drawing the borders first. Therefore they are not noticing the structure of rows and columns and they are not using the language of **unitising**, for example, *five fours*

Second stage: Students structure one of the dimensions (rows or columns), but not both.

Children at this stage talk about repeated addition in the rows or columns but are unable to consider both simultaneously.

Third stage: Students become able to use the square units as indicators of how many rows and how many columns, but they still struggle to understand that one square can simultaneously represent a column and a row.

Array cards

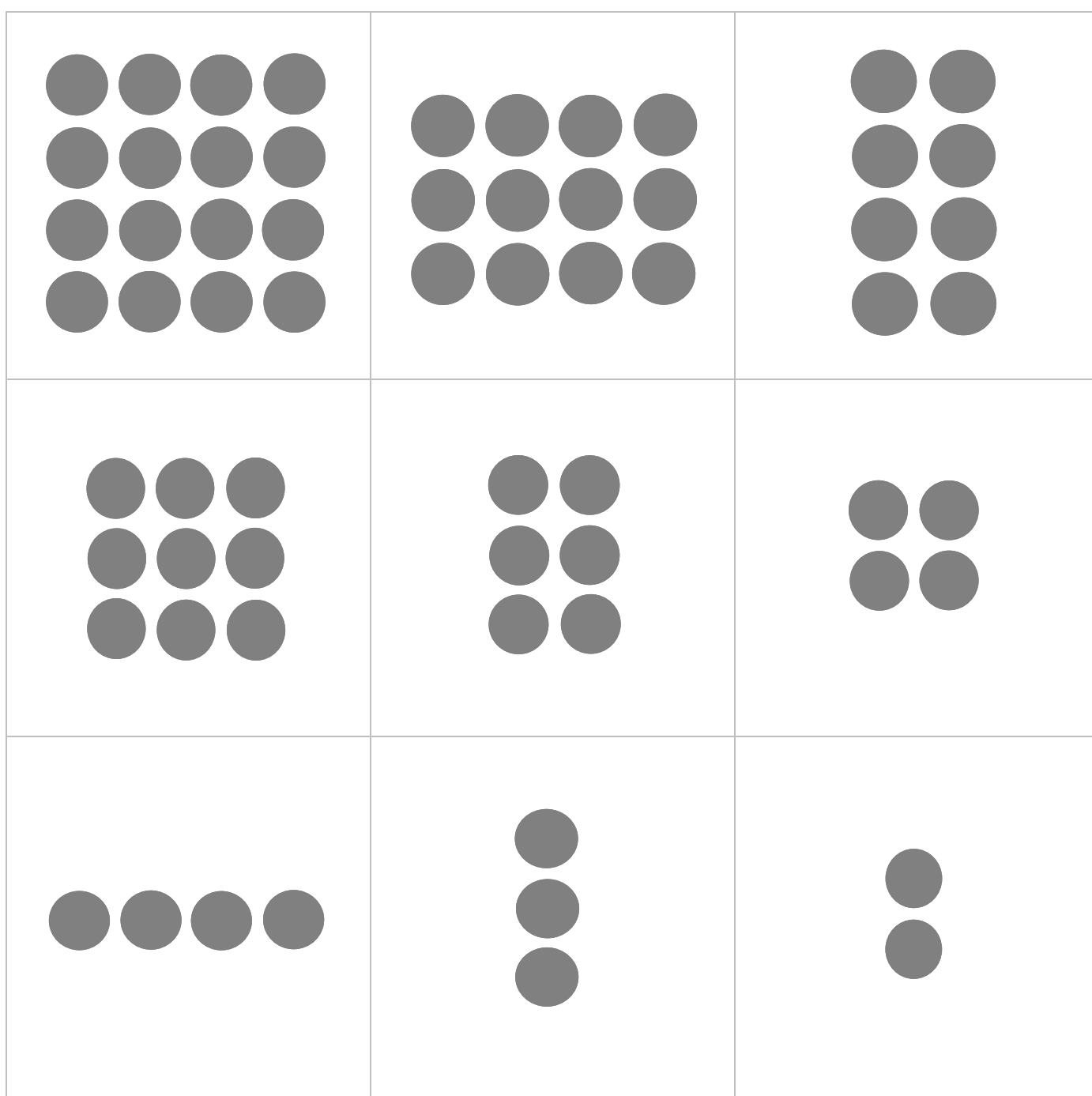
Year 1 onwards

Give children lots of opportunities to make and explore arrays: through making them with counters, by printing with circular objects or by pressing circular objects into play-dough.

Use the array cards below as flash cards. Ensure that the children can already subitise.

Let the children see the card for a short time and ask them to visualise the pattern of dots and say how many. Play snap. Play pairs.

To begin with children will need to count the number of dots but with practise they will learn to recognise the pattern of dots.



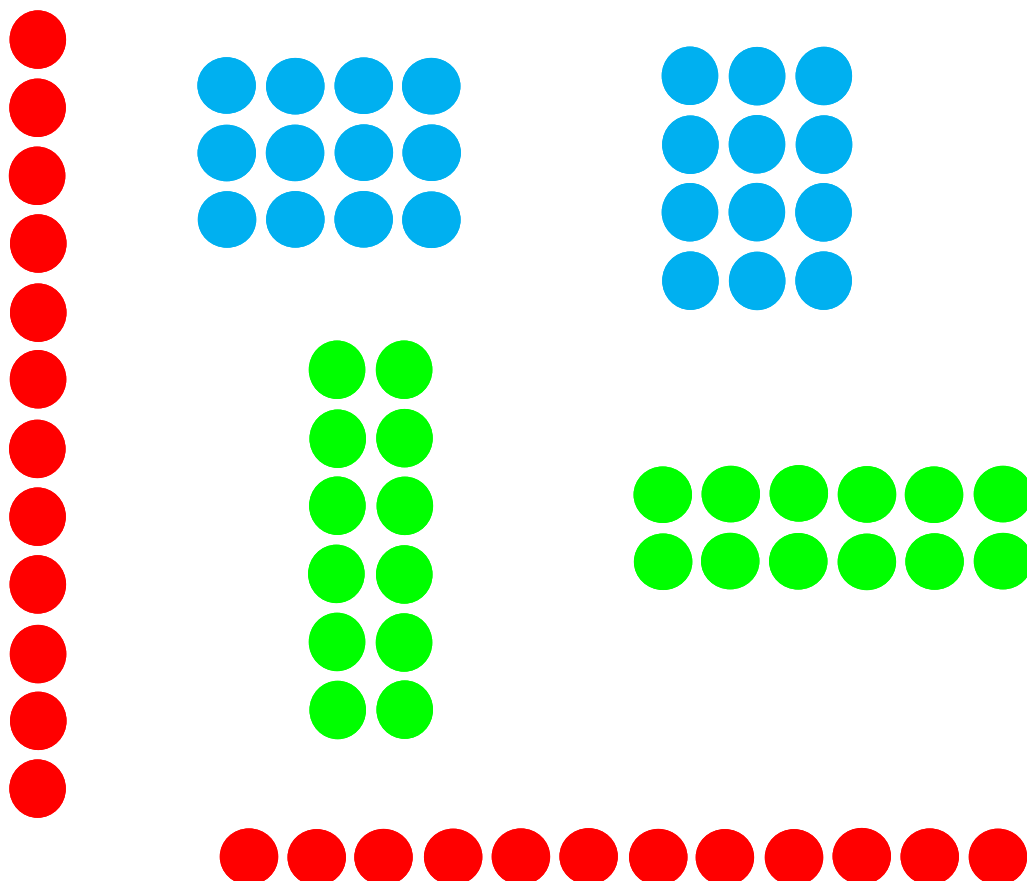
Arrays

Year 2 onwards

Here are 12 counters.

Arrange them in equal rows.

Try different ways



Investigate arrays

Explore making arrays with different numbers

Here are 6, 9, 12, 13, 15, 16, 17, 18, 20, 24, 25, 36... counters

Find different ways to arrange them in equal rows.

What do you notice? What's the same? What's different?

Annotate your arrays - for example '3 fours' or 3 rows of 4, 3 lots of 4, 3×4

Representing multiplication situations as arrays

- Match a situation (a context) to an array - a situation told verbally
- Match a situation to an array and to a calculation
- Match an array to a calculation and vice versa
- Make links between arrays
- Create or draw an array for a situation
- Create or draw an array for a calculation
- Draw an array for a situation or calculation and find how many by partitioning - (children should not be counting in ones—they should be encouraged to skip count or use the distributive property - partitioning)
- Make links between different arrays - seeing one array in another and talk about and record the multiplicative relationships
- Make links between area and perimeter and between volume and surface area - “how many different –sized boxes can you make to hold thirty-six Christmas ornaments and how much cardboard would be needed for each one. Making links to packaging (D&T). Three-dimensional arrays allow children to investigate the associative property for multiplication in a context that makes sense to them”.

(Fosnot and Dolk 2001 p.45)

Ensure that children experience a range of multiplication situations that involve:

- Equal groups - i.e. cookies on plates
- Arrays
- Constant rate -(money, speed, measures)
- Area
- Combination problem - find all possibilities i.e. 4 ice-cream flavours with 3 different toppings or clown hat problems
- Scaling - find a ribbon that is 4 times as long as this one

Big ideas in multiplication

The Commutative Property

The Distributive Property

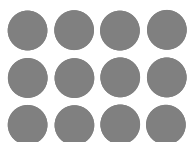
The Associative Property

It is important that these properties of multiplication are seen (noticed) and explored in two-dimensional arrays (commutative property) or with three-dimensional boxes - the (associative property).

See www.mathsisfun.com

Year 2 onwards

The Commutative property



$$4 \times 3 = 3 \times 4$$



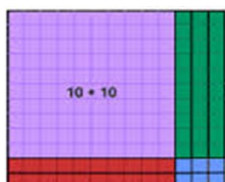
The Distributive property - Year 4 onwards

Realising that 9×5 can be solved by adding 5×5 and 4×5 or any combination of groups of five that add up to nine groups (for example 6×5 and 3×5) - this involves understanding about the structure of the part/whole relationship involved.

When using the distributive property, learners have to think about how to decompose the whole in groups.

The distributive property is also a central organising idea in mathematics; it is the basis for the multiplication algorithm with whole numbers.

$$12 \times 13 = (2 \times 3) + (2 \times 10) + (10 \times 3) + (10 \times 10)$$

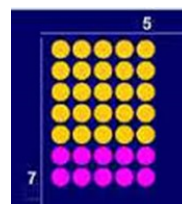


Grid method 12×14

	10	2	14
10	100	20	
4	40	8	

$100 + 20 + 40 + 8 = 168$

$$7 \times 5 = (5 \times 5) + (2 \times 5)$$

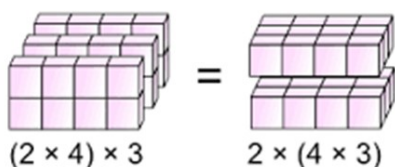


And in algebra $(x + 1)(a + 4) = x^2 + 4x + x + 4$

Also that $7 \times 8 = (8 \times 8) - 8$

\times	x	$+1$
x	x^2	$+1x$
$+4$	$+4x$	$+4$

The Associative property - Year 4 onwards



$$(2 \times 4) \times 3$$

$$2 \times (4 \times 3)$$



$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

Associative Property

$$(6 \times 4) \times 2 = 6 \times (4 \times 2)$$

$$24 \times 2 = 6 \times 8$$

$$48 = 48$$

Fact boxes

Children create fact boxes relevant to their understanding and to develop the range of facts they know and can use fluently.

In Year 2 children are learning the multiplication and division facts for the 2, 5 and 10 multiplication tables and are developing their fluency in applying these. They are also learning to count in steps of 2, 3 and 5 from 0 and in 10s from any number

In Year 3 children are learning the multiplication and division facts for the 3, 4 and 8 multiplication tables - therefore they should be fluent with the 2, 5 and 10 multiplication tables.

In Year 4 children are learning the multiplication and division facts for the 6, 7 and 9, 11 and 12 multiplication table - therefore they should be fluent with the 2, 3, 4, 5, 8 and 10 multiplication tables

1	→	4
2	→	8
4	→	16
10	→	40
5	→	20
20	→	80

Create and begin to explore your first fact box

Year 2 onwards

Year 2 example:

Create a fact box with a group of children

Use a multiplication table that the children know fluently.

Create the fact box with the children, using this format.

Write the facts one at a time.

Children are usually happy with one lot of 5 and two lots of 5 but when the teacher writes 4 lots of 5 next, they often query this and ask why it is not three lots that go next. At this point it is important to talk to the children about the difference between what they have already experienced, that is 'times tables' which list all the multiples of 5 and a fact box which lists some of the multiples of 5.

Ask the children if it is possible to use the fact box to calculate 3 lots of 5 (children will know what 3 lots of 5 are but, we are asking them to identify how they can derive this fact from the information in the fact box). This can cause 'cognitive conflict' for children who have learnt the multiplication facts for 5 by rote. Here we are asking children to apply what they know in a more complex way. We are asking them to notice a connection - to notice that 2 lots of 5 added to 1 lot of 5 is equal to 3 lots of 5. Representing this as an array would be useful. However, if children cannot, fairly quickly, make this connection, then they are not ready to explore fact boxes.

Children will know the product of 4 lots of 5 but, we also ask them how they might use another fact in the box - double 2 lots of 5.

By this stage the children will have taken on board that a fact box includes some of the multiples of 5 and therefore will be happy that the next two multiples are 10 lots of 5 and 5 lots of 5. Again it is important to ask the children to notice connections between these two facts and other facts in the fact box.

Ask children to write out this fact box for themselves - use A3 paper and felt pens.

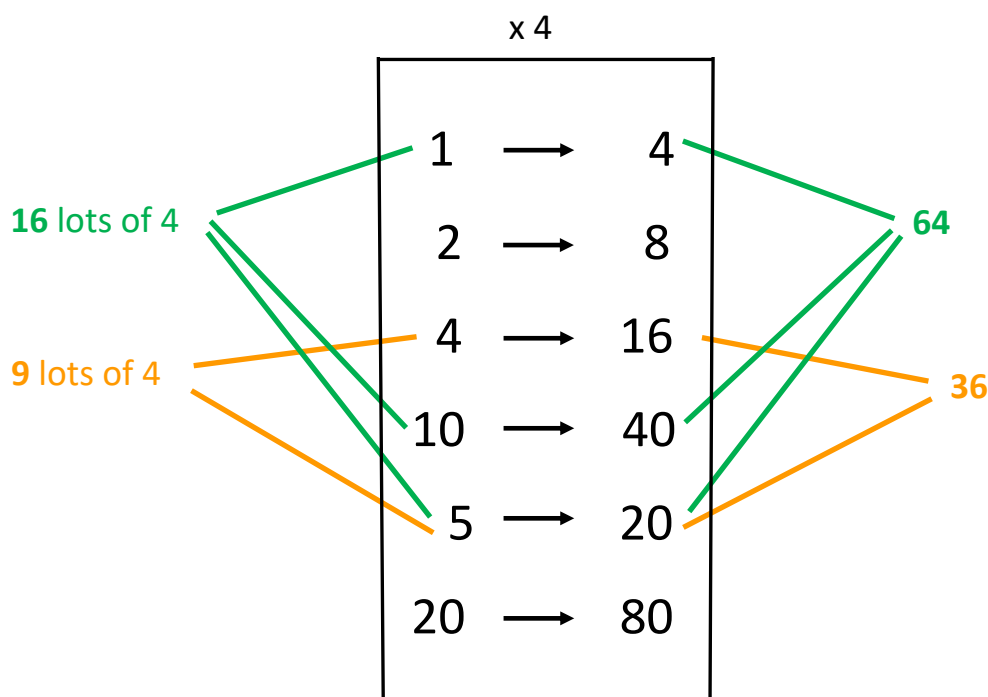
Ask the children to use the fact box to find the product of 7 lots of 5.

Once children are confident, ask them to work independently to find 9 lots of 5, 11 lots of 5, 13 lots of 5, 19 lots of 5 ...

1	→	5
2	→	10
4	→	20
10	→	50
5	→	25
20	→	100

Explore multiplication using a fact box

Year 3



Children are learning that **9 lots of 4** are equal to 5 lots of 4 plus 4 lots of 4

Also ask children to represent 9 lots of 4 (4×9 , 9×4) with counters or by drawing an array.

Arrange the counters to show that 5 lots of 4, plus 4 lots of 4 is equal to 9 lots of 4.

Use this fact box to calculate:

3 lots of 4, 3×4

7 lots of 4, 7×4

9 lots of 4, 9×4

Annotate your fact box

Ensure that colour is used mathematically

On the next page is an example of Jens's work.

Notice his effective use of colour.

How could we express how we calculated 16×4

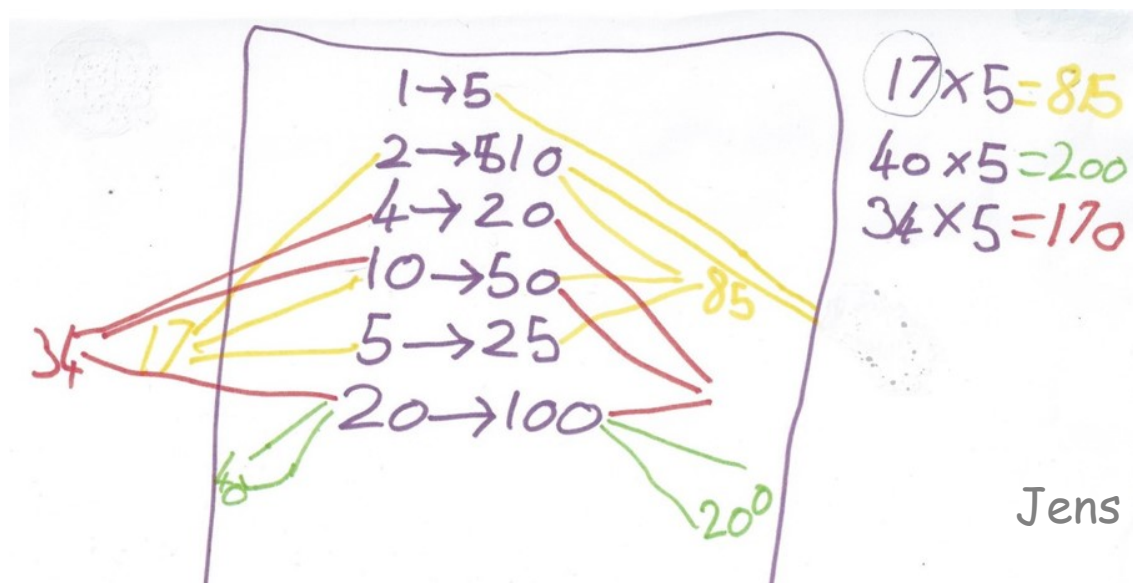
$$16 \times 4 = (10 \times 4) + (5 \times 4) + (1 \times 4)$$

The distributive property (law)

Making connections

Here is an example of Jens's work - **Year 2**

Notice his effective use of colour.



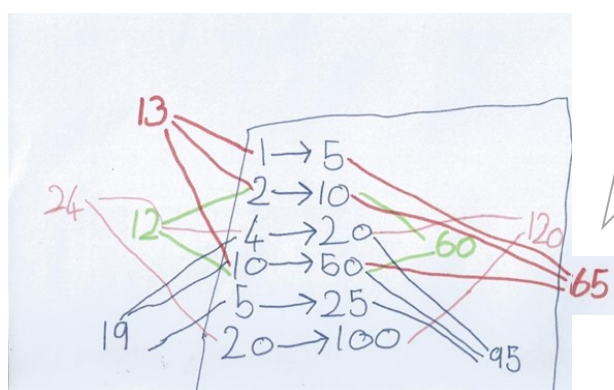
Spend time exploring fact boxes, developing children's 'number sense' and making the links to known facts (times tables), prior to calculating the answers to questions.

What do you notice?

What else can you calculate using your fact box?

What is the next fact you would add to your fact box? Why?

**Make connections between
multiplication and division**



How many fives are there in 65?

Jens combines the products to make a total of 65 and then combines the number of fives to answer this question.

The recording is clear because of Jens's use of colour

Comparing fact boxes - making connections

Year 4 onwards

What do you notice?

1	→	7
2	→	14
4	→	28
10	→	70
5	→	35
20	→	140

1	→	17
2	→	34
4	→	68
10	→	170
5	→	85
20	→	340

Can you calculate the product of 17×6 , if you know the product of 7×6 ?

Why is the product of 17×4 forty more than the product of 7×4 ?

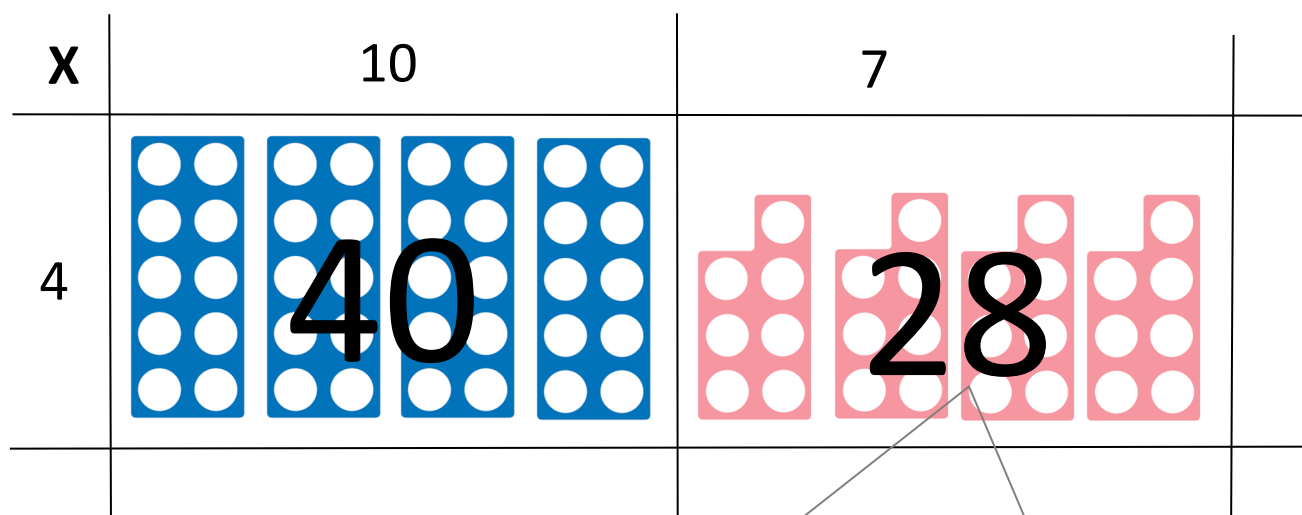
17×4 is 68. How can you use this fact to calculate the product of 27×4 ?

Multiplication - using Numicon to model the grid method

Applying mental calculation fluency

Year 3

$$17 \times 4$$



Children know this is 28 because:

- They put the Numicon shapes together and work out the total using the tens, without counting
- they are fluent at doubling - doubling 7 to get 14 and then doubling 14
- they know their multiplication facts for the 7 times table (*year 4*)

Children combine the partial products mentally to find the product of 17×4 .

Later children might record this as

$$\begin{array}{r}
 17 \times 4 \\
 \hline
 10 \times 4 + 7 \times 4 \\
 40 + 28 = 68
 \end{array}$$

Multiplication - an array representation

Area model

Year 3 onwards

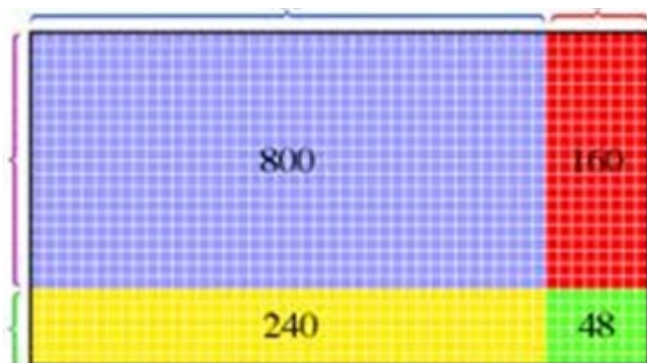
Area model - Partial products

Annotate this array to show how the **distributive property** has been used and to show the **partial products**



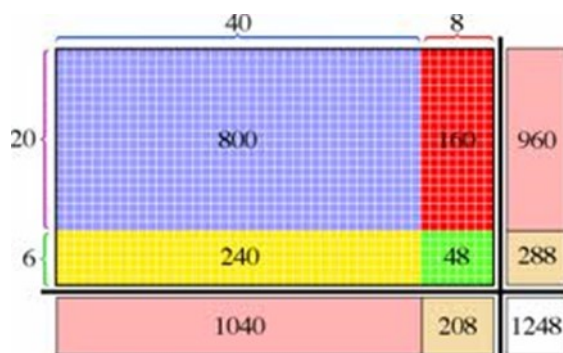
Annotate this array to show how the **distributive property** has been used and to show the **partial products** of the rows or the columns.

Then find the **product** of 48×26



How will you add the partial products (the rows or the columns) to find the product of 48×26 ?

Why did you make that decision?

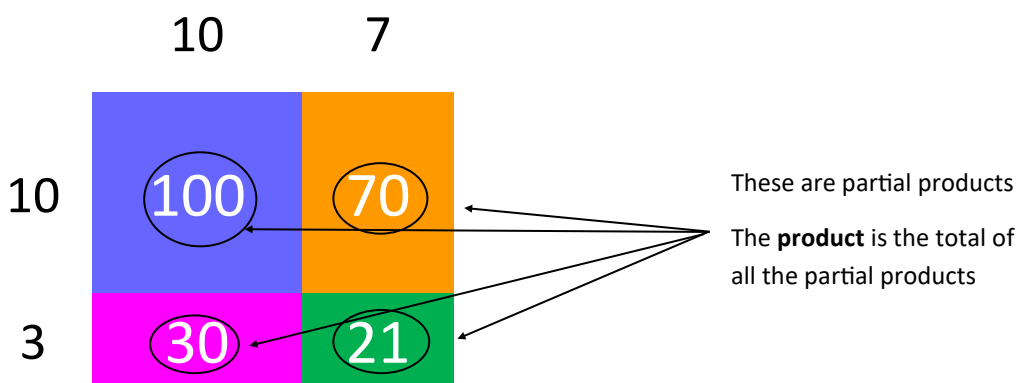
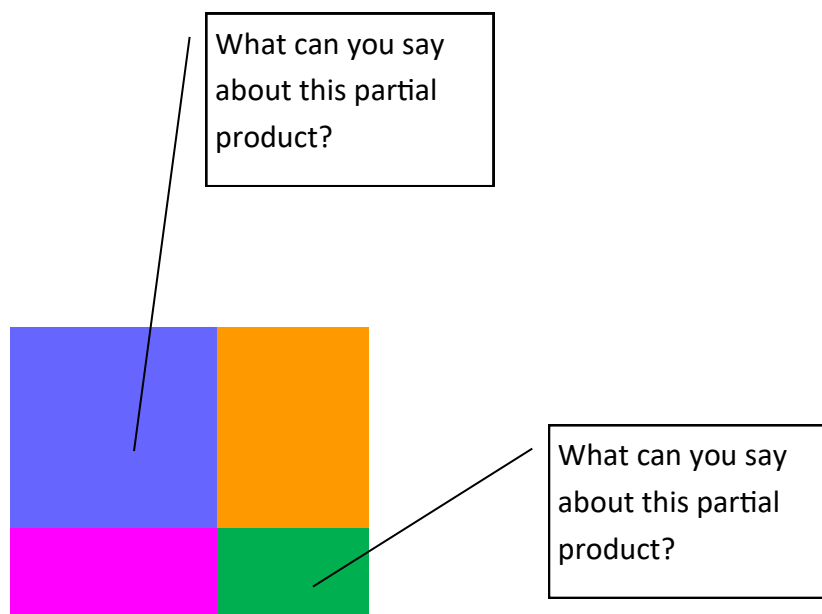


Multiplication - The Area model

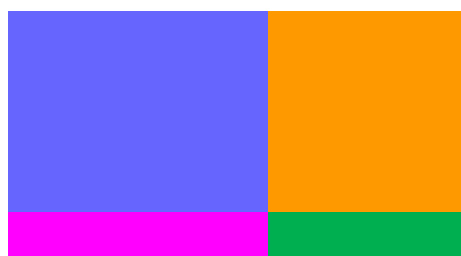
The boxes in the grid are proportional

Year 3 onwards

Area model - Partial products



x



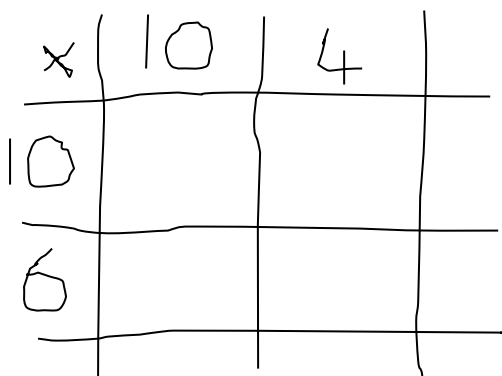
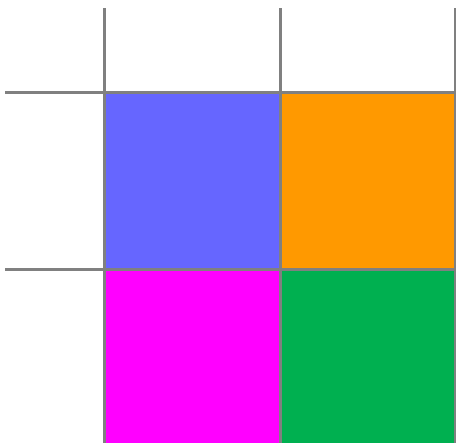
What calculation could this array represent ?

Why?

Multiplication—The geometric model

The size of each of the 'boxes' in the grid is the same

Year 3 onwards



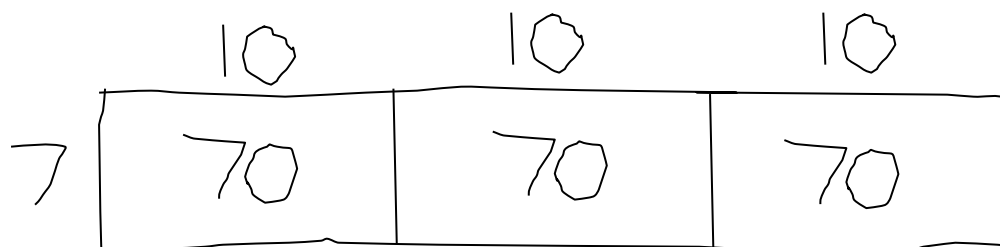
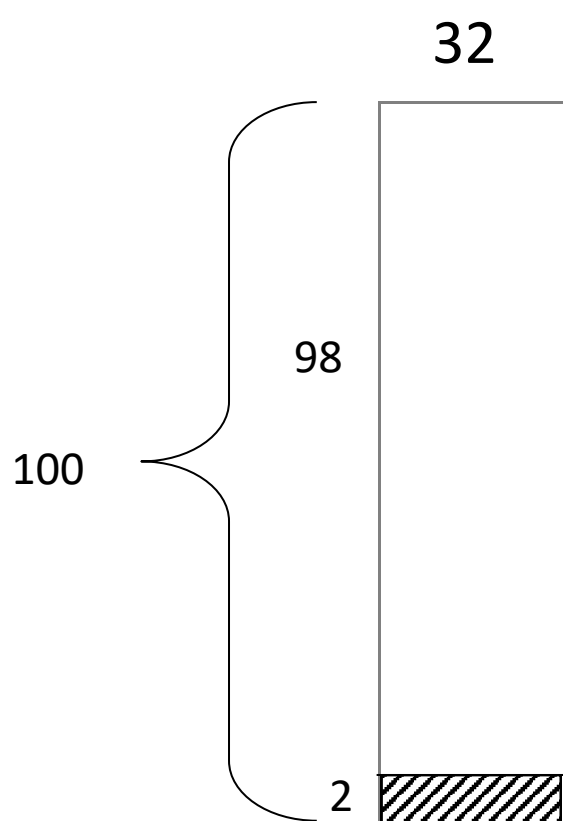
Open arrays

“A powerful tool to think with”

Years 5 and 6

$$98 \times 32 = (100 \times 32) - (2 \times 32)$$

$$= 3200 - 64 = 3136$$

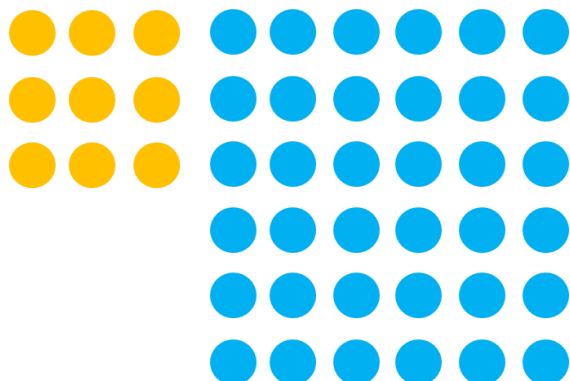


$$30 \times 7 = 3 \times (7 \times 10)$$

Looking at the structure contained within the representation

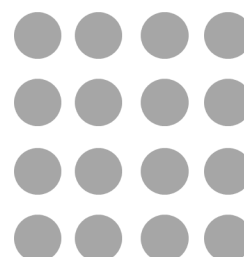
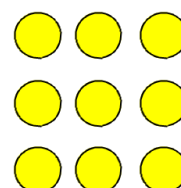
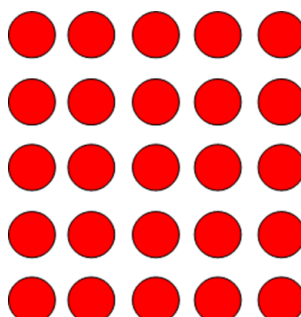
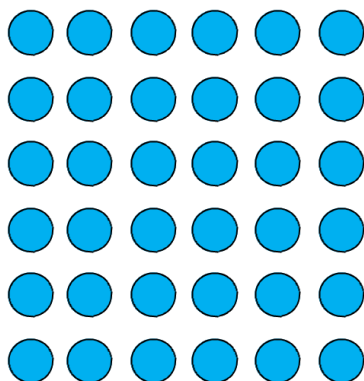
Noticing and making connections

Years 5 and 6



4 lots of 9 has the same value as 36
 4 lots of 9 has the same value as 6 lots of 6
 36 is equal to 4 lots of 3^2
 6^2 is equal to 4×3^2
 $3^2 \times 4 = 36$
 A quarter of 36 is 9
 $3^2 = 36 \div 4$
 $6^2 = 3^2 \times 4$
 $6^2 \div 4 = 3^2$
 $\frac{3}{4}$ of 36 = $3 \times 3^2 = 3^3$

Make connections



Procedural Fluency

Noticing and making connections

Years 5 and 6

$$\begin{array}{l} 8 \times 9 \\ 9 \times 6 \\ 17 \times 5 \\ 15 \times 7 \\ 37 \times 4 \end{array}$$

This is an example of a random set of questions

When we ask children (or adults) which question they will do or did first, they will usually say that they started with the first question, then 'did' the second question and so on until all have been completed. There is nothing to notice! The focus is on whether the product is right or wrong.

Now look at the set of questions below and ask children what they notice.

What do you notice?

This is an example of a designed set of questions

Stick this sticker into the middle of your page and annotate it to show what you have noticed.

$$\begin{array}{l} 8 \times 7 \\ 13 \times 7 \\ 16 \times 7 \\ 4 \times 7 \\ 32 \times 7 \\ 2 \times 4 \times 7 \end{array}$$

Here are some of the things we want children to notice and talk about:

- The multiplier is the same.
- One calculation involves three numbers and two multiplication signs - (the associative property of multiplication).
- Which calculation they will do first and why?
- Which order they will do the calculations and why?
- What other calculations you would add to this set and why?
- Which calculation you did last and why? ...

Sets of questions to develop reasoning about multiplica-

Years 5 and 6

$$14 \times 4$$

$$14 \times 2$$

$$7 \times 8$$

$$12 \times 3$$

$$12 \times 6$$

$$24 \times 6$$

$$7 \times 5$$

$$14 \times 5$$

$$3.5 \times 10$$

$$9 \times 8$$

$$18 \times 8$$

$$18 \times 16$$

$$16 \times 3$$

$$4 \times 3$$

$$8 \times 6$$

$$8 \times 9$$

$$24 \times 3$$

$$12 \times 3$$

$$8 \times 0$$

$$8 \times 8$$

$$4 \times 4$$

$$6 \times \frac{1}{2}$$

$$6 \times 2$$

$$6 \times 1$$

$$6 \times 12$$

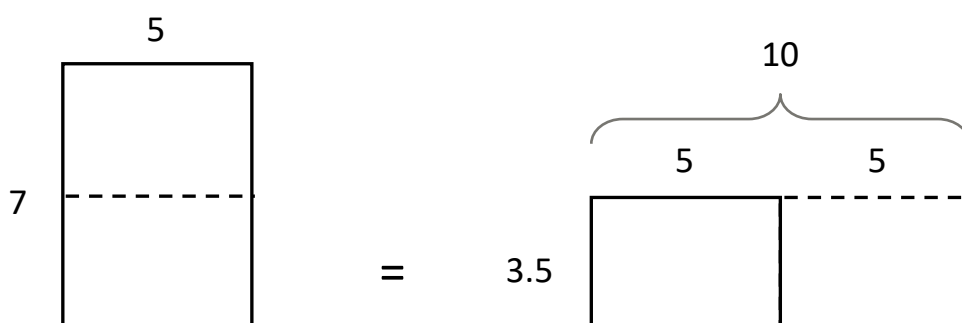
$$12 \times 12$$

$$6 \times 24$$

Ask pupils to add 2 more calculations to each set and to justify why they belong in the set.

Ask pupils to draw representations to prove why some calculations in a set have the same product.

Here is an example - You can draw an array to show that $7 \times 5 = 3.5 \times 10$



Multiplication

Formal written methods

“Algorithms - a structured series of procedures that can be used across problems, regardless of the numbers - do have an important place in mathematics. After students have a deep understanding of number relationships and operations and have developed a repertoire of computation strategies ...

“Algorithms should not be the primary goal of computation instruction... Using algorithms, the same set of steps with all problems, is antithetical to calculating with number sense. Calculating with number sense means that one should look at the numbers first and then decide on a strategy that is fitting - and efficient. Developing number sense takes time; algorithms taught too early work against the development of good number sense. Children who learn to think, rather than to apply the same procedures by rote regardless of the numbers, will be empowered. They will not see mathematics as a dogmatic, dead discipline, but as a living, creative one. They will thrive on inventing their own rules, because these rules will serve afterwards as the foundation for solving other problems.”

Fosnot. C.T. and Dolk. M. (2001) Young mathematicians at work: *Constructing Multiplication and Division*. Heinemann p.102.

Short multiplication - use the language of column value

Year 3 onwards
When pupils are
fluent with the
grid method

$$12 \times 7 = 84$$

$$\text{factor} \times \text{factor} = \text{product}$$

Short multiplication

Two-digit x one-digit

No carrying

$$\begin{array}{r} 22 \\ \times 4 \\ \hline 88 \\ \hline \end{array}$$

Short multiplication

Carrying from tens into a new place -
hundreds

$$\begin{array}{r} 32 \\ \times 4 \\ \hline 128 \\ \hline \end{array}$$

Short multiplication

Carrying units

$$\begin{array}{r} 23 \\ \times 4 \\ \hline 92 \\ \hline 1 \end{array}$$

Short multiplication

Carrying from units to tens and from
tens to a new place - hundreds

$$\begin{array}{r} 33 \\ \times 4 \\ \hline 132 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 266 \\ \hline 5 \end{array}$$

Short multiplication

$$12 \times 7 = 84$$

$$\text{factor} \times \text{factor} = \text{product}$$

Year 4 onwards

Multiple two and three digit numbers by a one-digit number

Year 5 onwards

Multiply numbers up to 4 digits by a one or two-digit number

Short multiplication

Three-digit x one-digit

$$\begin{array}{r} 116 \\ \times 3 \\ \hline 348 \\ \hline 1 \end{array}$$

Also include calculations which involve using numbers with the same three digits - for example

$$116 \times 3$$

$$161 \times 3$$

$$611 \times 3$$

Numbers with a zero

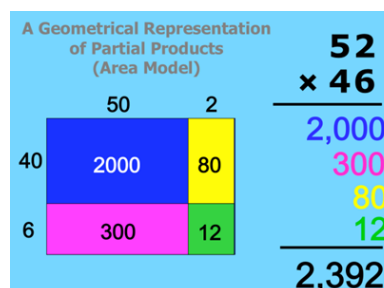
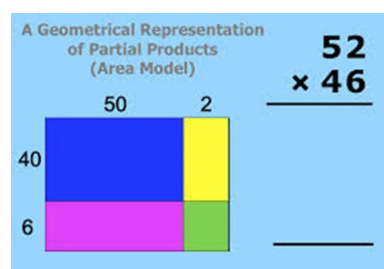
$$\begin{array}{r} 307 \\ \times 4 \\ \hline 1228 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 370 \\ \times 4 \\ \hline 1480 \\ \hline 2 \end{array}$$

Expanded vertical method leading to long multiplication.

Make links with the grid method

Two-digit x two-digit



Long multiplication

Two-digit x two-digit

$$\begin{array}{r} 52 \\ \times 46 \\ \hline 2080 \\ 312 \\ \hline 1 \\ 2392 \end{array}$$

Long multiplication

Year 5 onwards

$$12 \times 7 = 84$$

$$\text{factor} \times \text{factor} = \text{product}$$

Year 5 onwards

Multiply numbers up to 4 digits by a one or two-digit number

Expanded multiplication

Two-digit x two-digit

$$\begin{array}{r} 43 \\ \times 27 \\ \hline 21 \\ 280 \\ 60 \\ 800 \\ \hline 1161 \\ 1 \end{array}$$

Long multiplication

Two-digit x two-digit

Starting with the tens digit

$$\begin{array}{r} 43 \\ \times 27 \\ \hline 860 \\ 301 \\ \hline 1161 \end{array}$$

Long multiplication

Two-digit x two-digit

Starting with the ones digit

$$\begin{array}{r} 43 \\ \times 27 \\ \hline 301 \\ 860 \\ \hline 1161 \end{array}$$

Expanded multiplication

Three-digit x two-digit

$$\begin{array}{r} 124 \\ \times 27 \\ \hline 28 \\ 140 \\ 700 \\ 80 \\ 400 \\ 2000 \\ \hline 3348 \\ 1 \quad 1 \end{array}$$

This is not efficient. A grid method would be more appropriate.

Long multiplication

Three-digit x two-digit

Starting with the tens digit

$$\begin{array}{r} 124 \\ \times 27 \\ \hline 2480 \\ 868 \\ \hline 3348 \\ 1 \quad 1 \end{array}$$

Long multiplication

Three-digit x two-digit

Starting with the ones digit

$$\begin{array}{r} 124 \\ \times 27 \\ \hline 868 \\ 2480 \\ \hline 3348 \\ 1 \quad 1 \end{array}$$

Appendix

Array cards

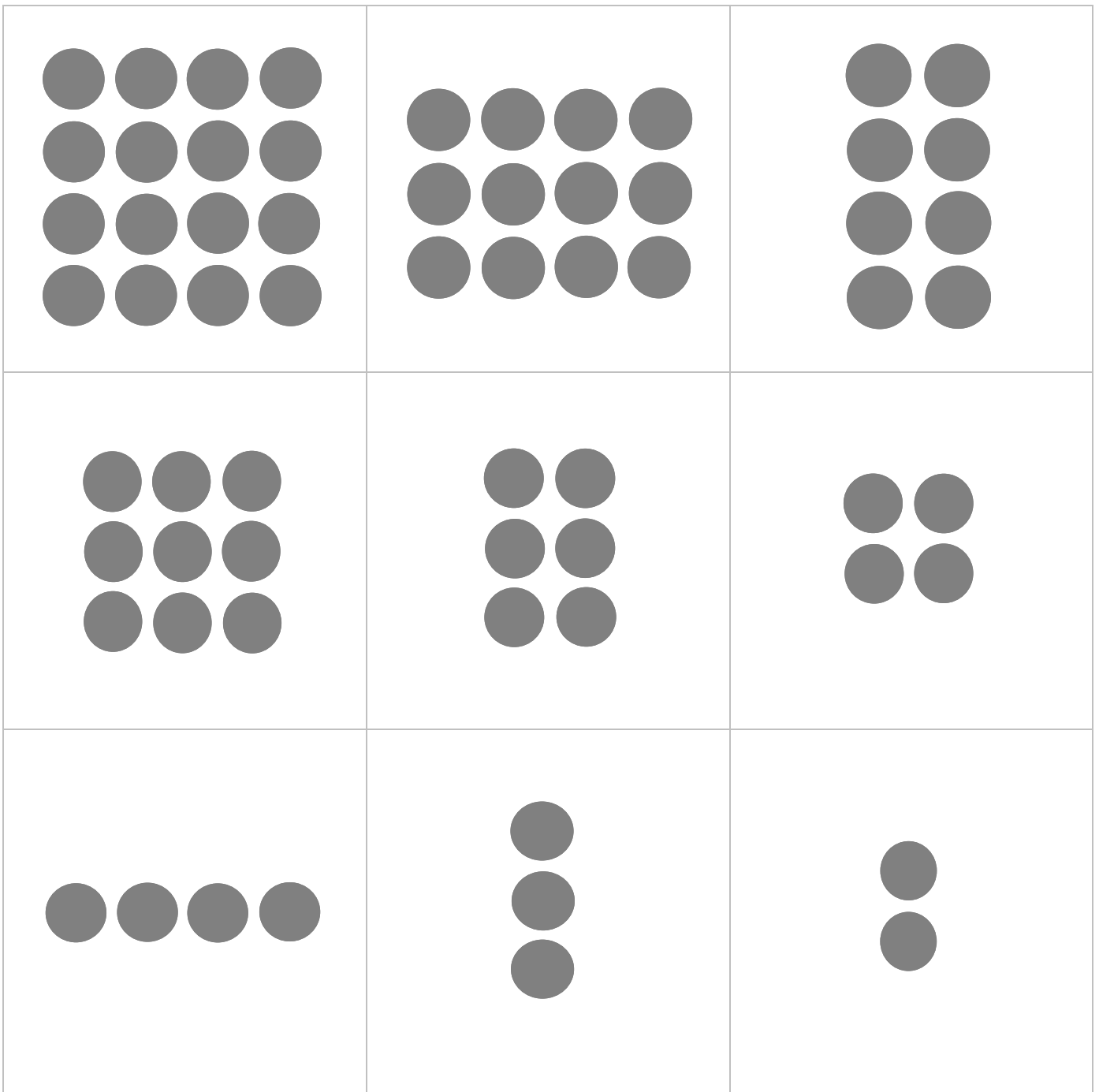
For pupils with Dyslexia use the blue arrays printed onto yellow card.

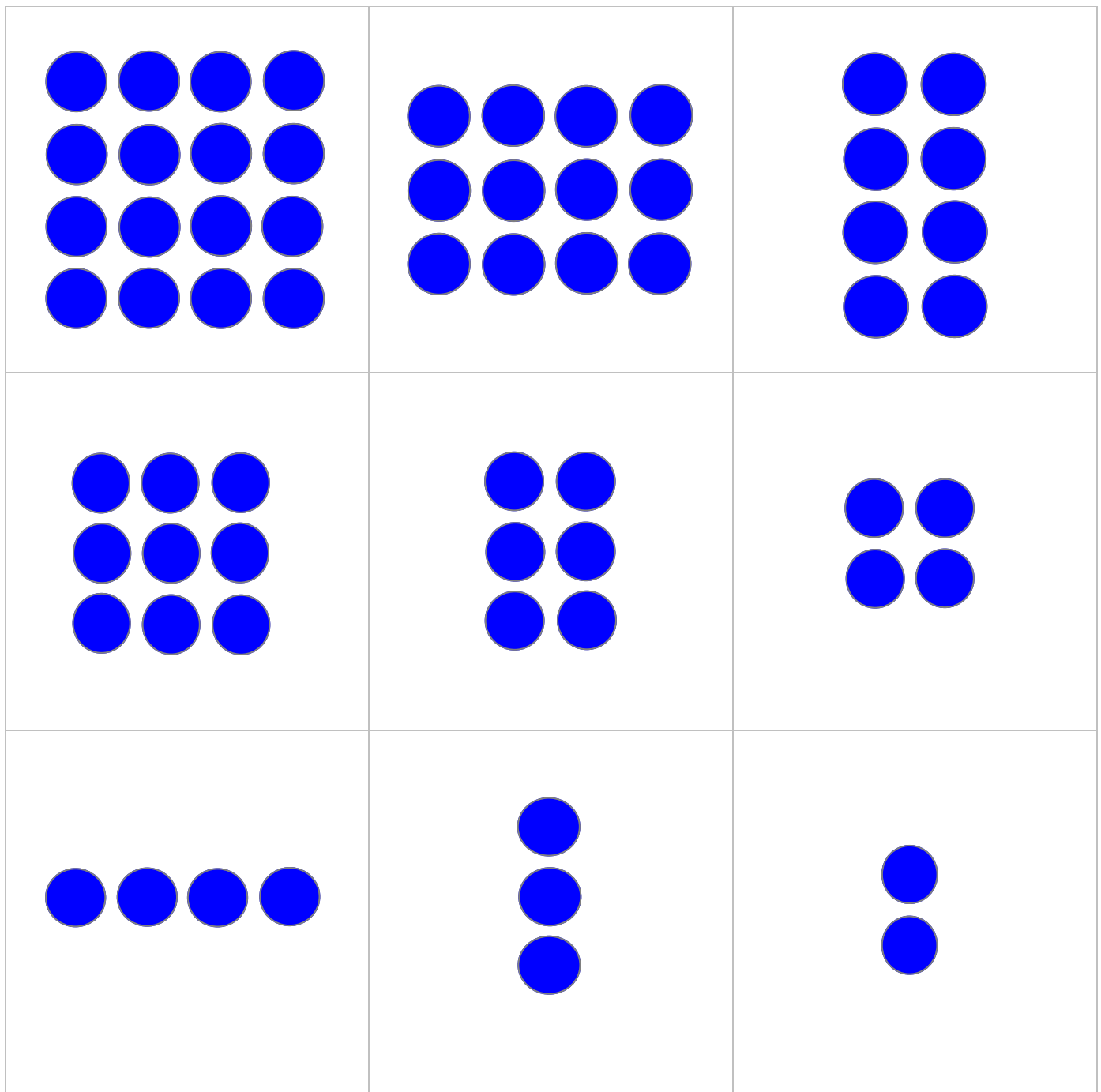
The Grid Method

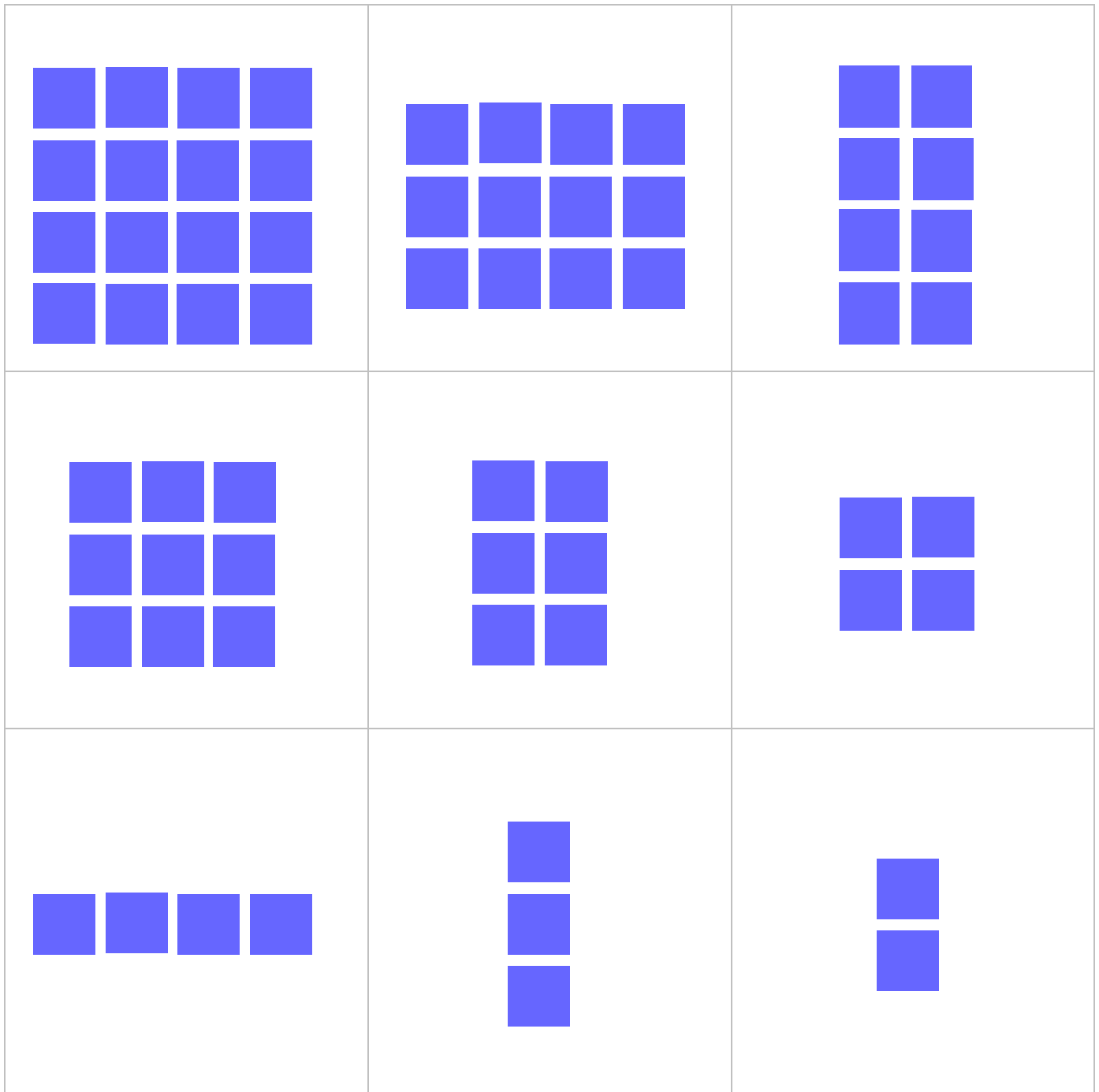
Activities to develop understanding of multiplication in years 3, 4 and 5.

These build fluency in using the grid method prior to moving onto the formal written methods of short and long multiplication.

These activities provide opportunities for pupils to apply their growing knowledge of known multiplication facts.







The 'Grid Method'

Years 3 and 4

Year 3

- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, **including for two-digit numbers times one-digit numbers**, using mental [and progressing to formal written methods].

Year 4

- Multiply and divide numbers mentally drawing upon known facts.
- **Multiply two-digit and three-digit numbers by a one-digit number** [using formal written layout].

The 'Grid Method' is an informal method for multiplication where children use and apply their mental calculation skills:

Below identifies the fluency that pupils need to understand prior to being taught the 'Grid Method' for multiplication:

- **Place value** - multiplying a multiple of ten by a single-digit number.
- **Times tables** - because the box in the lower right-hand corner of the grid is always the **application of times tables knowledge** (or deriving a fact if it cannot be recalled - *this does not include counting in steps of*) Therefore this method requires fluency with times tables as appropriate for the year group.
- **Addition** - adding a 'string' of numbers together efficiently using informal methods with jotting by gathering the **partial products** together efficiently to find the **product** OR using the formal written method if appropriate.
-

Match the calculation to the correct grid
and justify why you think it is correct.

18×7

12×5

17×5

31×6

13×6

15×3

24×6

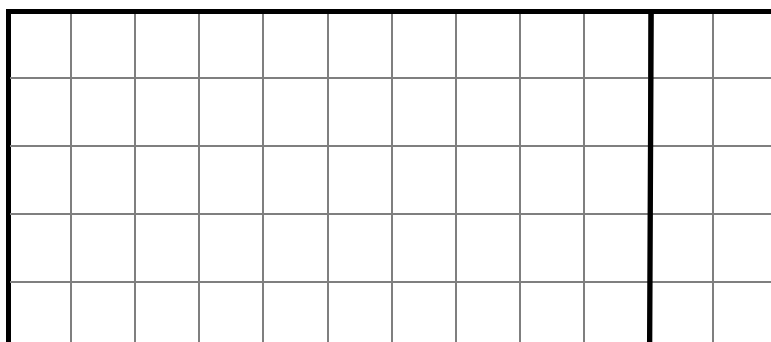
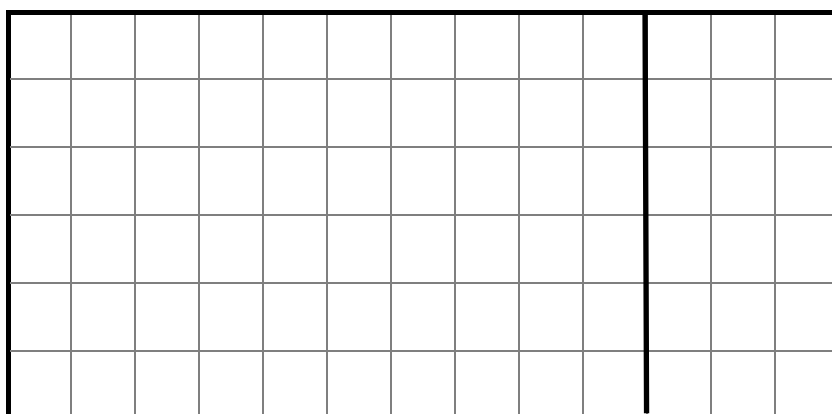
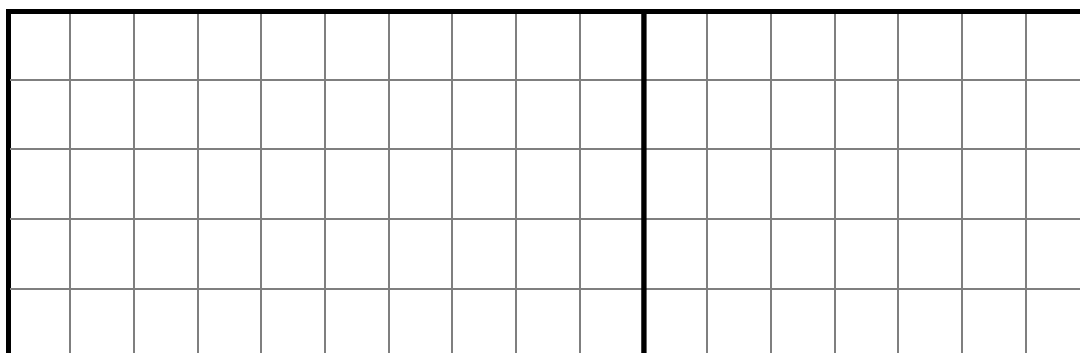
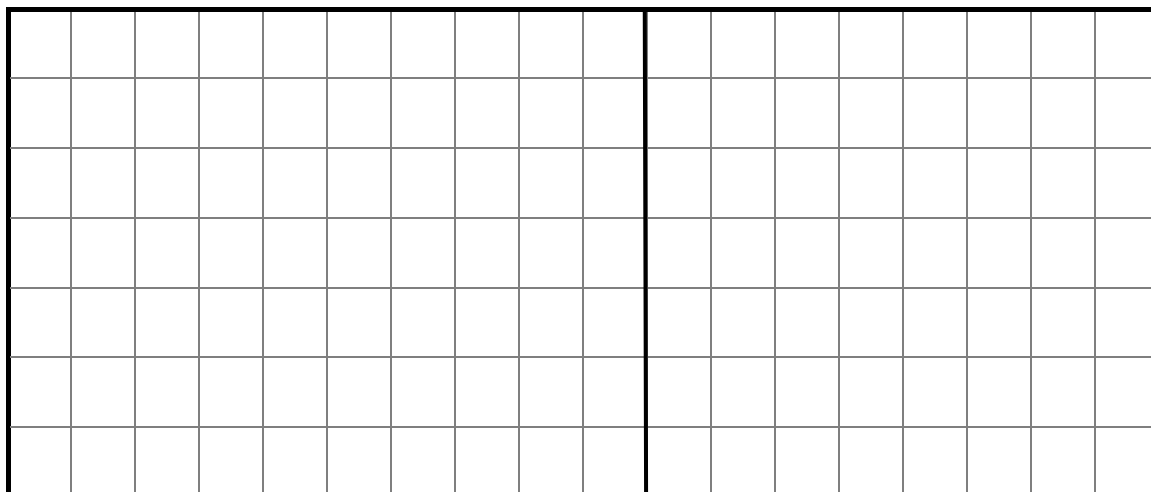
19×4

23×8

14×9

27×3

23×6



[illegible][illegible][illegible]

The 'Grid Method'

Year 5

The 'Grid Method' is an informal method for multiplication where children use and apply their mental calculation skills:

Year 5

- Multiply and divide numbers mentally drawing upon known facts.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Multiply numbers **up to 4 digits by a one-digit numbers and two-digit numbers** [using a formal written method, including long multiplication for two-digit numbers].

Below identifies the fluency that pupils need in place prior to being taught the 'Grid Method' for multiplication:

- **Place value** - multiplying a multiple of ten by a single-digit number.
- **Times tables** - because the box in the lower right-hand corner of the grid is always the ***application of times tables knowledge*** (or deriving a fact if it cannot be recalled - *this does not include counting in steps of*) Therefore this method requires fluency with times tables as appropriate for the year group.
- **Addition** - adding a 'string' of numbers together efficiently using informal methods with jotting by gathering the ***partial products*** together efficiently to find the ***product*** OR using the formal written method if appropriate.

Match the calculation to the correct grid
and justify why you think it is correct.

NOTE: There is not a grid for each calculation - you will need to draw a grid for the calculations that do not have one already drawn.

$$16 \times 12$$

$$117 \times 37$$

$$23 \times 26$$

$$143 \times 23$$

$$25 \times 21$$

$$738 \times 13$$

$$24 \times 18$$

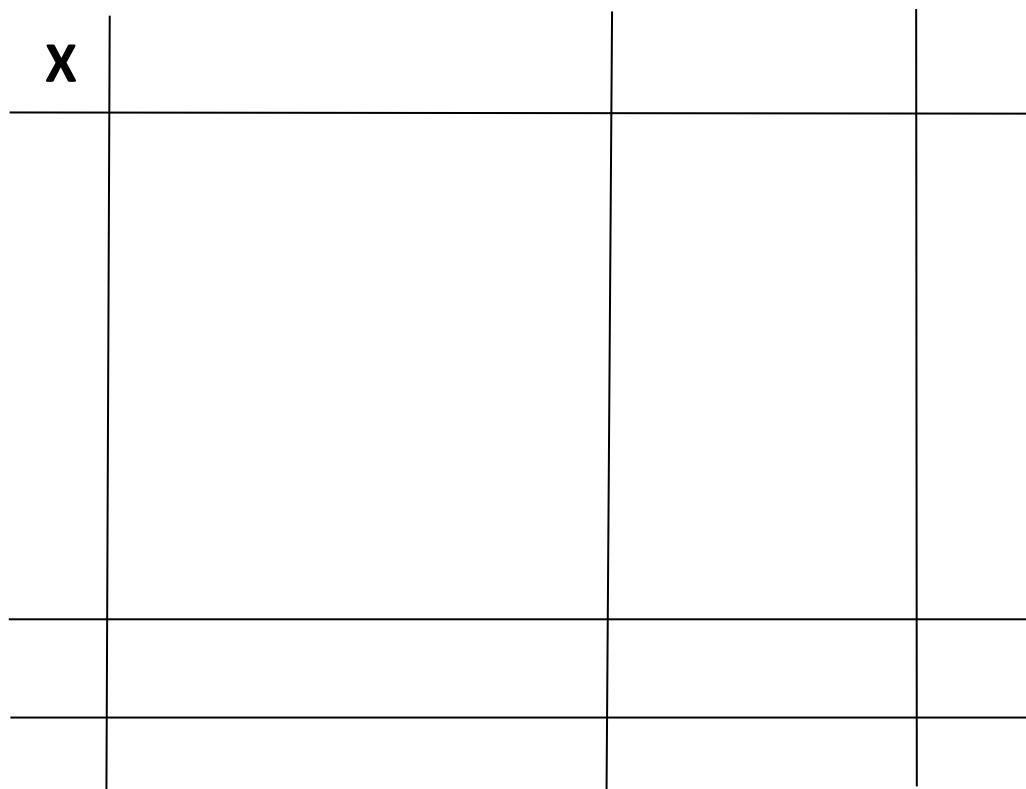
$$587 \times 58$$

$$19 \times 15$$

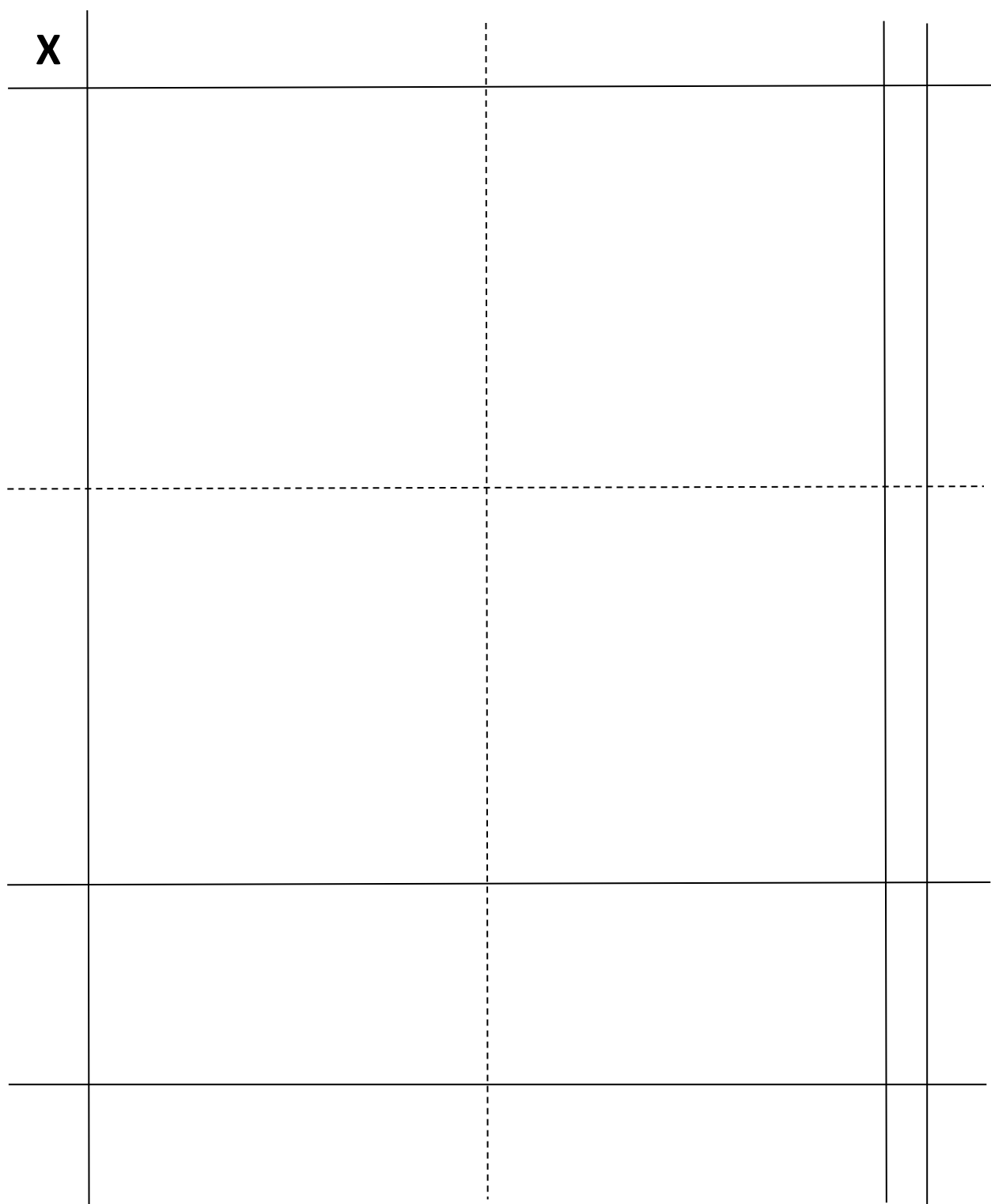
$$3062 \times 16$$

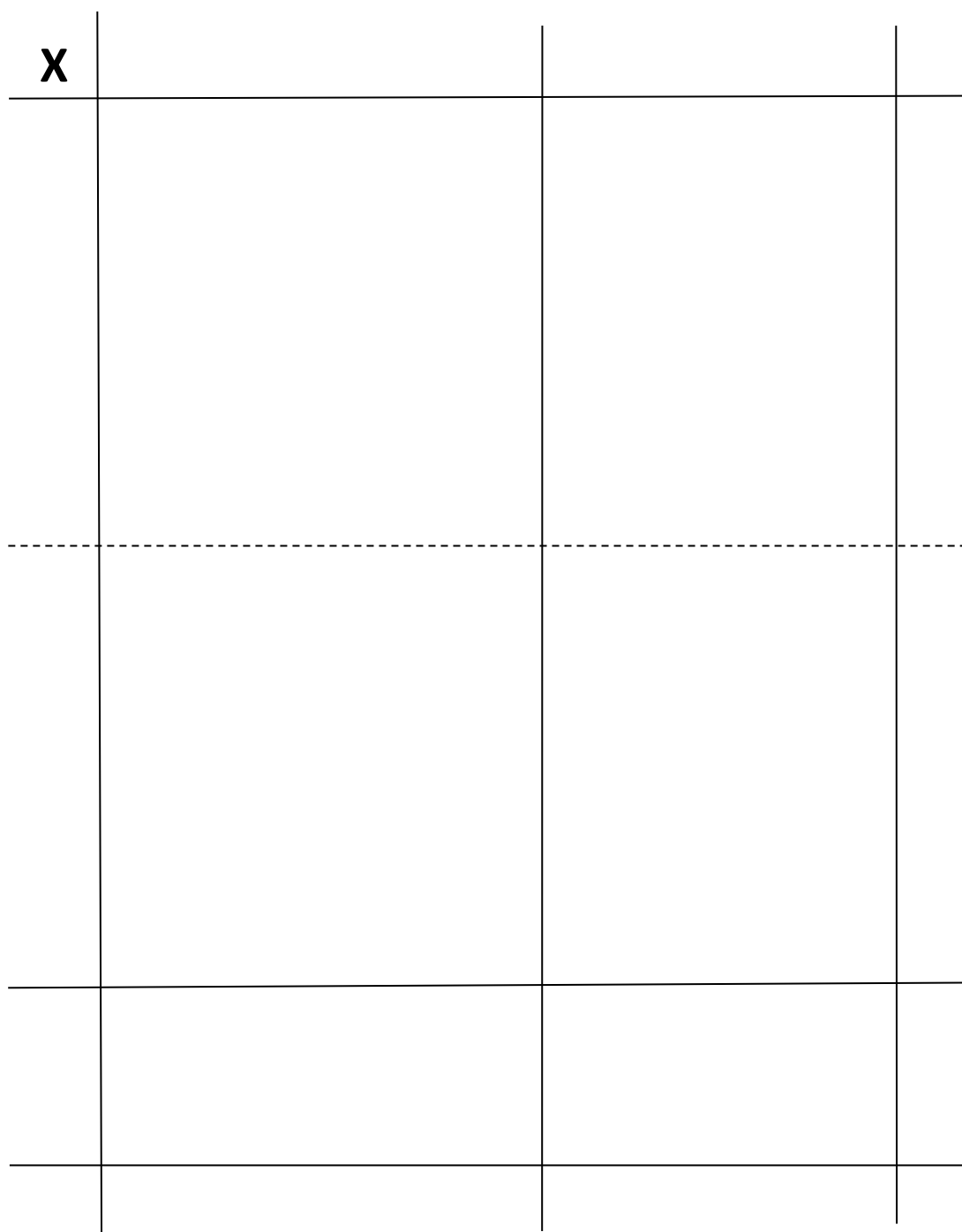
$$18 \times 13$$

$$1803 \times 27$$



X





X

X
